The Fracturing of Certainty: Mochizuki and the Crisis of Mathematical Universalism

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The abc conjecture

An **abc triple** is a triple of positive integers a, b, c such that $a + b = c$ and $\gcd(a, b) = 1$.

The **quality** of an abc triple is

$$q(a, b, c) = \frac{\log(c)}{\log(\prod_{p|abc} p)}.$$

(Here $\prod_{p|abc} p$ means the product of the prime divisors of abc. Thus if $abc = 6 \cdot 3^2 \cdot 5$, then $\prod_{p|abc} p = 15$.)

Conjecture (The abc conjecture) For every $\epsilon > 0$, there are only finitely many abc triples such that $q(a, b, c) > \epsilon$.

Joseph Oesterle

David Masser

Shinichi Mochizuki

(DD, 2016)

1. The End of the Universal Language

For centuries, mathematics was considered the "universal language"—the one domain where truth was not subject to culture, geography, or interpretation. A proof was either right or wrong, and a mathematician in Kyoto could speak with perfect clarity to one in Bonn. The controversy surrounding Shinichi Mochizuki's Inter-universal Teichmüller (IUT) theory and his alleged proof of *the abc conjecture* has shattered this illusion. It suggests that mathematics, like literature or religion, can branch into "sects" that no longer share a common vocabulary.

The a + b = c conjecture is a proposition in number theory, stating, roughly, that when a + b = c and a, b, c share no common prime factors, the product of the distinct primes dividing a, b, c is usually not much smaller than c, so cases where c is much larger are very rare (Fesenko, 2015; Scholze and Stix, 2018; Yamashita, 2017; Mochizuki, 2024; Joshi, 2025a, 2025b).

2. The Burden of Communication vs. The Burden of Study

At the heart of the conflict lies a radical disagreement over the responsibility of the genius.

The Western Critique: Figures like Peter Scholze argue that the "burden of communication" lies with the author. If a proof cannot be condensed into a logic that the existing community can verify, it fails as a social contract.

The Kyoto Perspective: Mochizuki and his proponents argue that the "burden of study" lies with the community. They contend that the complexity of modern number theory has reached a point at which a "quick summary" is impossible, and that Western skepticism is a form of intellectual laziness or "short-circuiting" of a new, valid logic.

3. The Rise of Mathematical Nationalism

The divide has taken on an uncharacteristic nationalist tone. The Japanese mathematical establishment's decision to publish the proof in a journal edited by Mochizuki himself—despite the loud protests of international experts—was seen by many as a "circling of the wagons." This raises a troubling philosophical question: Can a mathematical truth be "local"? If the Japanese mathematical community accepts a proof and the Western community rejects it, the very definition of "objective truth" begins to look more like "institutional consensus."

4. The Limits of Human and Artificial Verification

The saga highlights a terrifying milestone in human knowledge: the point where a theory becomes too dense for the collective human mind to audit.

The Human Limit: If only a dozen people in the world claim to understand a proof, and they all reside within the same institutional orbit, can we ever truly call it "verified"?

The AI Illusion: While we look to so-called "artificial intelligence"(AI) as a potential tie-breaker, the "IUT crisis" exposes the limits of current AI. To verify the

proof, a human must first translate Mochizuki's hundreds of pages of new definitions into a machine-readable language (formalization). This process itself requires a level of human understanding that currently does not exist in a consensus form.

5. Conclusion: A New Era of Mathematical Limbo

The Mochizuki controversy might be the first of many. As we push the boundaries of abstract thought, we risk creating "intellectual islands" where truth is determined by loyalty to a specific framework rather than universal verification. The abc conjecture remains a "ghost theorem"—treated as a foundation in some parts of the world, but as a myth in others. It serves as a reminder that even in the most logical of fields, pure mathematics, truth is still a human construct, vulnerable to the same tribalism and pride that define the rest of our history.

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