

Supertasks and the Consistency of the Real Numbers: Reply to Jéssica Milaré/Supertarefas e a consistência dos números reais: Resposta a Jéssica Milaré

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Men at work down under, performing supertasks, circa 1981 (Filmaffinity US, 2023)

In this short essay we respond to five objections made by Professor Jéssica Milaré to an earlier essay (Smith, Smith, & Stocks, 2023) in which we presented a dilemma to the effect that if one holds to the coherence of the idea of a supertask, of performing an infinity of actions in a finite time, as most philosophers of mathematics do to counter Zeno's paradoxes, then an argument can be given that the real numbers are inconsistent. This is meant to be a *reductio* of the idea of the coherence of the concept of supertasks, rather than a proof of the inconsistency of the real numbers. Professor Milaré thinks that we are barking up the wrong gum tree, as the saying goes on our continent (i.e., Australia) and perhaps the beautiful country of Brazil as well, Brazil being now the world's top producer and exporter of eucalyptus wood and pulp (GFC, 2023).

Each of Professor Milaré's objections is quoted below in italics, followed by our reply to that objection. Before we get into the heat of professional academic "Mortal

Kombat," however, we give thanks from the abyss of our dark hearts to Professor Milaré for taking the time to read it and respond to two philosophical nobodies from The Land Down Under, "Where beer does flow and men chunder" (Men at Work, 1981).

Objection 1

JM: There is an inconsistency in the reasoning of this text. You question the validity of the concept of "supertask", but you apply that notion in a simplistic manner for mathematics, without formally defining what means to execute a supertask.

Reply to Objection 1

JWS & SJS: The concept of a supertask is repeatedly defined in the literature referenced in the essay, and it should not be necessary to reinvent the wheel with every essay. Roughly, a supertask is a countably infinite sequence of actions, performed in a finite amount of time, to paraphrase the article on supertasks in the *Stanford Encyclopedia of Philosophy* (Manchak & Roberts, 2022). So, the notion is defined by its proponents. There is no inconsistency in applying the concept of a supertask, as defined by its proponents, to then show its incoherence. That, of course, is known as a *reductio ad absurdum*, or *redução ao absurdo*, argument. Surely mortal humankind, even those capable of performing supertasks, can be required do no more?

Objection 2

JM: For example, how do you define mathematically the "supertask" of writing "999..."?

Reply to Objection 2

JWS & SJS: We mathematically define this supertask just as the proponents do in dealing with the Thomson lamp paradox, or with any of Zeno's paradoxes, by describing the object in natural language. First write the sign "9", then "99" then "999" and "so on." Thus "999 ..." is simply an infinite concatenation of "9." That "and so on" the "..." are favourites of mathematicians, so surely we can use it too?

Objection 3

JM: Why does writing a sequence of digits necessarily produce a value, and what value is that, how do you compute or produce or define on terms of the axioms of real numbers?

That is similar to simply writing a sequence of letters and claiming it is a meaningful word without defining its meaning.

Reply to Objection 3

JWS & SJS: This sort of objection could be used, and in fact is used by ultra-finitists like Norman Wildberger, against the existence of infinity in mathematics, where as we referenced in the paper, he shows that representation of real numbers by infinite decimals has insuperable conceptual problems. But, writing a sequence of digits does indeed produce a number, which is the idea behind numbers being represented by infinite decimals, although as Wildberger shows, the orthodox mathematicians are wrong about this. Moreover, there is an important difference between “999...” and a random assortment of, say, Portuguese letters.

Objection 4

JM: *Note that, when we write (I’ll use comma as decimal separator)*

0,999... = 1,000...

Both sides are informal notations for a limits of sequences defined by induction. To prove that equality, you need to consider a mathematical definition of limit (e.g. the standard epsilon-delta definition of limit) and then write the formal version of that notation.

That, of course, depends also on the set of axioms of real numbers that you take into consideration—if, e.g., you consider intuitionistic mathematics, the axioms are different. But you are considering classic logic with standard real numbers, therefore the equality is a theorem.

Reply to Objection 4

JWS & SJS: Yes, absolutely, the numbers 0.999... and 1.000... can be represented by limits of sequences, as well as many other notions. They can also, from a formalist/conventionalist/nominalist viewpoint, be represented as concatenations of signs. That is the position taken in our essay. And we have no objection to taking $0.999... = 1.000...$ as a theorem; which we in fact do. Now let’s use the supertask magic to move the decimal an infinite number of times to the right. If we are not permitted to do this, then why should *any* supertask be accepted, all of them being much vaguer than this simple action? A lamp that switches on and then off an infinite number of times in a finite time? Moving through an infinity of points in a finite time? Go figure!

Objection 5

JM: *On the other hand, the sequence “999...” is not well defined on hyperreal numbers. There are many ways to define that, but in any event your whole argument fails:*

E.g. if can you take the equality

$$0,999... = 1,000...$$

and multiply it by a fixed infinite hyperinteger [sic] that is a power of 10. Note there are many infinite hiperintegers [sic] so you have to fix one of them. In any case, after that, you would have, essentially, the following equality:

$$999...999,999... = 1000...000,000...$$

but that is an abusive (ambiguous) notation, because the order of magnitude of the power of ten disappeared, thus the notation is insufficient.

In any case, the difference would still be zero.

Reply to Objection 5

JWS & SJS: With all due respect, we think that the argument from the hyper-reals is irrelevant, and question-begging, since we are taking the theorem in the real numbers, $0.999... = 1.000...$ and applying a supertask to it. It does not matter if $999...$ is not well defined on the hyper-reals, since we are not saying that it is a hyper-real.

Therefore, the argument presented in (Smith, Smith, & Stocks, 2023) survives the criticisms of Professor Milaré, and we heartily thank her again for taking the time to read and respond to it.

REFERENCES

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