Creativistic Philosophy: Exploring the Limits of Formalization, #12[1]—Formalizable Systems and Proteons.

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In previous installments, we have explored some of the limits of algorithms. We have delved into the part of mathematics and theoretical computer science called computability theory, e.g. looking at topics like the "Halting Problem". This branch of mathematics deals with questions of what is computable and what is not.

We have seen before that the notion of algorithms is just another form of the concept of a formal theory. In a formal theory, a set of statements can be derived by some rules of inference. An object is formalizable if it can be described completely in terms of a formal theory. This means that all true statements about it can be derived in

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[2] The left-hand picture shows the central part of the GEO600 gravitational wave telescope in Ruthe near Hanover, Germany. It stands here for the "hard" sciences which attempt to yield completely formalized descriptions of aspects of reality. GEO600, to be closed down end of 2026, is too small to actually detect gravitational waves, but many of the technologies used in larger telescopes were developed here. The right-hand picture shows the abstract sculpture "stardust" (2011) by Wilfried Behre, a sculptor from Hanover. The sculpture, a sparingly carved, partially polished block of stone in Behre's characteristic style, showing an unpolished circular area inside the polished surface on top and a polished circular area within the unpolished area on the left, stands on the GEO600 grounds, just a few meters from the main building where the first image was captured. It stands here for human culture which is not formalizable in principle. Both images © Andreas Keller 2026.

a single formal theory, or, in the language of algorithms, all its properties can be calculated using a single algorithm.

We have seen examples of mathematical objects that are not formalizable in this way:

- There is no way to generate exactly all algorithms for total functions by a single algorithm. Every algorithm that generates (enumerates) such algorithms must either be incomplete, i.e. produce only a true subset of the set of algorithms computing total functions, or will also produce some algorithms that do not produce total functions. So the set of all total functions is not formalizable.

- We have seen that the set of natural numbers cannot be described completely in terms of a single algorithm or formal theory. There are always more computable true statements about natural numbers than can be calculated by any single algorithm or be derived in any single formal theory.

- The set of halting algorithms, i.e. algorithms that halt for all their inputs, is not formalizable, as we have seen in the previous installment.

We can say that computability theory explores the limits of formalization and formalizability. So, we could call it "formalizability theory" instead.

Our examples of entities that cannot be formalized completely so far are from mathematics, but there are examples from physical reality as well.[3] In the exact sciences, there is a tendency to attempt to produce theories that allow us to formalize large areas of reality. There is a view held by some people, which we could call *scientism*, that a complete formalization of reality is possible and is the ultimate goal of science and that the resulting theory of everything could allow us to derive all truths from a single theory. In the light of the limits of formalization that we have found so far, such a program looks infeasible, however.

So let us put the naïve scientism aside and let us have a serious look into the matter of (non-)formalizability:

- There are objects in mathematics and in physical reality that can be formalized completely, i.e. formal theories or algorithms can be constructed that cover them completely. All properties of such objects can be computed and all true statements about them can be computed by a single algorithm or derived in a single formal

[3] I would like to discuss this point in more detail in a future installment. For example, a property of physical systems (whether or not the system has a so-called "band gap") has been identified for which it has been shown that it is undecidable. If another atom is added to such a system at a specific location (which is a normal physical process), this property generally cannot be predicted.

theory. I call such objects "formalizable systems" or simply "systems."[4] They behave in a systematic way, i.e. their structure and behavior is describable by a formalism. They are predictable and produce no surprises.

- There are other objects that cannot be formalized completely. Every formal description of such objects is incomplete. There are more true statements about them than can be derived in any single formal theory. I call such objects "proteons".[5] They behave in a (partially) unsystematic, or "protean" way.

- For a proteon, partial formalizations are possible which cover parts of it, but always leave an unformalized rest. This unformalized rest produces surprises, or anomalies, with respect to the given partial formalization.

- The partial formalizations can be extended, i.e. additional aspects of the whole can be described and these descriptions can be integrated into the formalization. However, in the case of a proteon, it is impossible in principle to eliminate the unformalized rest completely.[6]

- There cannot be an algorithm that automatically produces all possible extensions of a theory/algorithm describing a proteon and integrates them into the theory automatically in all cases, even if the extensions can, at least in some cases, be produced by a method like the "diagonal + 1" productive function. Moreover, there

[4] The term "system" is normally used for any kind of structured entity. However, a scientistic belief in the formalizability of everything seems to be behind this use of the word, so I propose to reserve the use of the term "system" to formalizable and thus "systematic" entities. Alternatively, we can speak about "formalizable systems" or "f-systems".

[5] For the motivation of this term, see (Keller, 2019), and specifically the sections on Odysseus and Proteus. Note that this is a term I have coined, so you are probably not going to find it elsewhere, unless somebody else has also adopted it.

[6] It might be that extensions cannot be produced in some cases at all, i.e. there might be proteons that have areas into which we cannot go at all. For example, take the set of real numbers. Mathematical formulas about real numbers contain variables. We assume that we can substitute any real number in the place of such a variable in order to make a statement about it, but this cannot work for some real numbers. Whatever notational system we can invent to describe subsets of real numbers, like the natural numbers, the rational numbers (fractions of natural numbers) or the numbers whose digits can be enumerated by an algorithm (which would be a finite text), or numbers for which a unique definition of any kind, a formula or term can be specified in some formal language, there is no way to invent a notational system to represent every real number uniquely by a finite text. The reason is that there are only countably many finite strings over any given finite alphabet, but uncountably many real numbers. So there must be vast subsets of the set of real numbers that cannot be reached at all by any notational system whatsoever. We cannot make mathematical statements about such numbers by substituting them for variables in formulas and one can ask if quantified statements, e.g. for-all-statements about all real numbers, make sense at all if substitution is not possible for all real numbers. Perhaps the whole notion of real numbers does not make sense. Going further into this strange and somehow murky area is beyond the scope of the current essay, but this matter points up fundamental limitations of mathematics and language.

can be other ways of producing extensions beyond fixed productive functions. We are entering here the area of creative processes, which are themselves not formalizable.

- If not only mathematics, but also physical reality contains proteons, a computable theory of everything must be impossible. A mathematical theory of everything could still be possible, but it would contain non-computable predicates and functions. Each body of mathematical knowledge to solve the equations occurring in such a theory would be incomplete. So there would always be cases where we would not know how to solve the equations or do the calculations. We would have to extend our mathematical tool kit again and again, adding additional axioms and rules of inference to our theory, to solve more of such cases, but there would always remain unsolved ones. This is actually the situation physicists are facing and the reason why they are using approximative "numerical" methods, simplified models, simulation models with limited exactness, etc. So, if we ever find a theory of everything, it may be of little practical use.

- If creative processes that produce knowledge about proteons are not formalizable, and if human cognition is creative in this sense, then humans, their societies and cultures are among the non-formalizable entities in physical reality. The scientific methodology of the "hard" sciences is then necessarily insufficient in the realm of humans. This is why academic disciplines that deal with human beings, their psychology, society, culture, and history use methods that are entirely different from those of physics. This is the main reason why the scientistic program of building all sciences according to the model of the hard natural sciences is not feasible.

- On the contrary, if non-formalizable entities even occur in physics, we can expect the "hard" sciences to change more into the direction of the human sciences.

The existence of proteons means that there can be no definitive, complete scientific methodology. Scientific methodology—or, more generally, human reason—cannot itself be fully formalized and is therefore itself a proteon. If it were formalizable, it would be limited and unable to be creatively adapted to a protean world. So, there must be an ongoing reflection on methods and on reason, in which methods are both justified and further developed, as part of all sciences, especially those that deal with proteons, as is constantly the case in the human sciences. This process of reflection on method and reason is (part of) philosophy.

REFERENCES

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