“This very sentence is illegible.” Phenomenological Paradoxes of The Logic of Legibility

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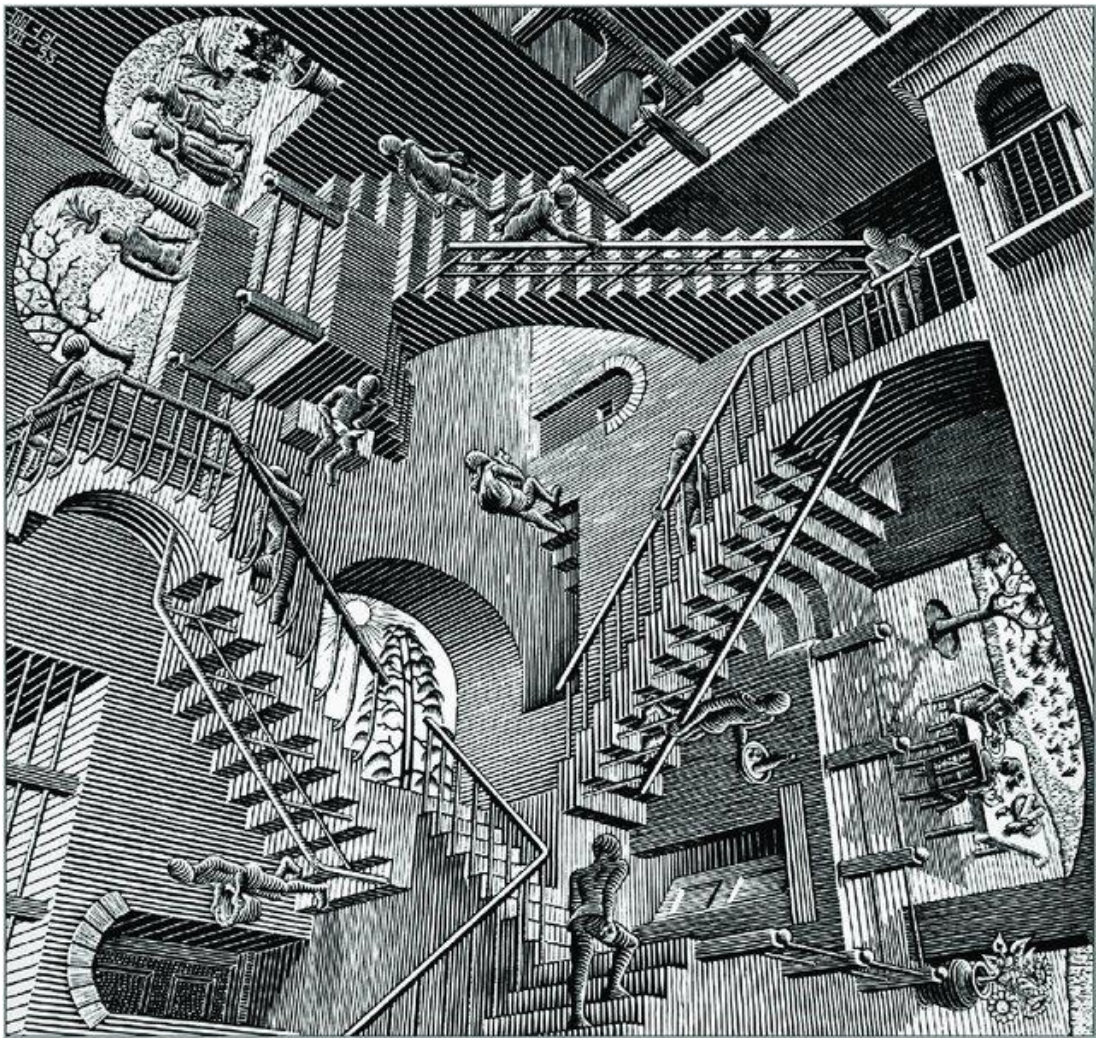

“Relativity,” by M.C. Escher (1953) (Valentino, 2020)

This very sentence is illegible.

For the purposes of this essay, I’ll assume that the universal Principle of Bivalence—i.e., that all meaningful indicative sentences or statements are either true or false, not both true and false (aka “truth-value gluts”), and not neither true nor false (aka

"truth-value gaps")—is true.[1] Now, if the first sentence of this essay is true, then it's illegible; but of course you, the reader of that very sentence, consciously read it, and, once you'd read this far into the *third* sentence of this essay—i.e., this very sentence—then you also self-consciously re-read the first sentence: so now you know with self-evidence that that the first sentence of this essay is legible and therefore false. But if the first sentence of this essay is false, then it's legible, and indeed you, the reader of that very sentence, consciously read it, and then self-consciously re-read it and thereby self-evidently confirmed that fact, which directly contradicts what the sentence says. Therefore, the first sentence of this essay is false if true and self-contradictory if false.

And that's a paradox, using this broad definition of "paradox":

> By "paradox" one usually means a statement claiming something which goes beyond (or even against) "common opinion" (what is usually believed or held). Paradoxes form a natural object of philosophical investigation ever since the origins of rational thought; they have been invented as part of complex arguments and as tools for refuting philosophical theses (think of the celebrated paradoxes credited to Zeno of Elea, concerning motion, the continuum, the opposition between unity and plurality, or of the arguments entangling the notions of truth and vagueness, credited to the Megarian School, and Eubulides of Miletus). Paradoxes—termed as *Insolubilia*—form also a substantial part of logical and philosophical investigations during the Middle Ages…. A terminological warning is in order. The word "antinomy" is used … as alternative to, and synonymous with, "paradox." Most paradoxes—but not all—involve contradictions; for such cases, we often use the word "contradiction" as well. (Cantini and Bruni, 2021)

Paradoxes that are most familiar to philosophers include Zeno's paradoxes, the Liar paradox, Kant's Antinomies of Pure Reason, Cantor's paradox, Russell's set-theoretic paradox, the true but undecidable and unprovable Gödel sentence, various epistemic paradoxes (Sorensen, 2022), and of course M.C. Escher's paradoxical visual figures, as per the image at the top of this essay, which also clearly show that paradoxes need not necessarily be encoded in language. But something that's unique about the paradox of illegibility that arises from the first sentence of this essay, however, is that it specifically *presupposes and uses the rational, conscious, self-conscious, intentional, and intersubjective or social* (after all, *I*, R.H., wrote the first sentence of this essay and then by means of posting this essay on academia.edu, I sent it out into the larger world, like a message in a bottle

[1] Actually, for reasons that I won't go into here, I think that the strong or universal Principle of Bivalence is in fact false, and that only a weak or minimal Principle of Bivalence—i.e., that *not all* but *only some* meaningful indicative sentences or statements are either true or false, not truth-value gluts, and not truth-value gaps (Hanna, 2006: esp. ch. 2; Hanna, 2015: ch. 4). But this won't have a direct bearing on what I want to argue here, since I think that the first sentence of this essay does in any case fall under the Principle of Bivalence, whether strong or weak.

floating on the seas of rational human life, but *you*, who are not R.H., as it were found and opened the bottle, and then read that very sentence) *act or process of reading* in order to activate the paradox. Correspondingly, I'll call it *a phenomenological paradox of the logic of legibility*.

> By "logic" I mean "the science of logic," and by that I mean the following:
>
> > First, the science of logic is about "schematizable" language, that is, orderly sequences of sentences linked together by fixed interpretations of the logical and nonlogical constants occurring in them. Second, and more precisely, the science of logic is about schematizable language in which some sentences are asserted and another sentence is asserted that in fact follows with necessity from the assumed truth of the asserted sentences. Schematizable language in which some sentences are asserted and another sentence is asserted that is held to follow from the others is an *argument*. The asserted sentences are the *premises* of the argument. The asserted sentence that is held to follow from the others is the *conclusion* of the argument. The fact (whenever it is a fact) that truth is necessarily preserved from the premises to the conclusion is the *validity* of the argument. And the necessary connection between the premises and conclusion of a valid argument is the relation of *consequence*. Thus logic is the science of the necessary relation of consequence. (Hanna, 2006: p. xv)

For me, the term "legibility" means the same as the term "readability," the very meaning of which of course presupposes and uses the rational, conscious, self-conscious, intentional, and intersubjective act or process of reading. Elsewhere I've proposed analyses of legibility and reading as follows:

> 1. A text T-in-L is legible if and only if T-in-L satisfies *the perceptibility condition*, *the syntactic condition*, and *the semantic condition*, and
>
> 2. all and only such texts-in-L have legibility. (Hanna, 2023a: p. 11)
>
> 1. A person P reads a text T-in-L if and only if P consciously or self-consciously at least minimally scans, at least minimally parses, and also at least minimally comprehends T-in-L, and
>
> 2. all and only such acts or processes are reading. (Hanna, 2023a: p. 14)

In that essay, I also explicate the meaning of the terms "text T-in-L," "the perceptibility condition," "the syntactic condition," "the semantic condition," "minimally scans," "minimally parses," and "minimally comprehends." For my purposes here, what's important is simply that *the logic of legibility presupposes and uses the phenomenology of reading*, and therefore the logic of legibility restricts the science of logic and its study of

the necessary relation of consequence to all and only schematizable linguistic texts that can potentially be read, or actually are read, by rational, conscious, self-conscious, intentional, and intersubjective animals like us.

Since the true but undecidable, unprovable, and also uncomputable *Gödel sentence* (Gödel, 1931/1967; Boolos and Jeffrey, 1989) is indeed a *legible text*, and since not only the Gödel sentence, but also *other* kinds of uncomputable texts, can be shown to be legible texts (Hanna, 2023b), then it follows that the scope of the logic of legibility exceeds *not only* the scope of the logic of provability in classical *Principia Mathematica*-style mathematical logic, *but also* the scope of the logic of computability and so-called artificial intelligence, or AI (Hanna, 2023c).

Do phenomenological paradoxes of illegibility that arise from sentences like the first sentence of this essay, *have* a solution, where by "a solution" I mean some constructive or critical analysis, or theoretical proposal, such that what's prima facie paradoxical, turns out to be logically and theoretically coherent, consistent, and unproblematic at the end of the day? Think here, for example, of Alfred Tarski's hierarchy-of-languages solution to the Liar paradox (Tarski, 1943, 1956). Or do phenomenological paradoxes of illegibility even *need* a solution in that sense?

My own view is that they don't even *need* a solution in that sense, because the existence of phenomenological paradoxes of illegibility *isn't* in fact a clear indication that there's something fundamentally amiss in the foundations of the logic of legibility, in the way that Russell's paradox, for example clearly shows us that there's something fundamentally amiss in the foundations of naïve set theory; or in the way that the Liar paradox clearly shows us that there's something fundamentally amiss in the foundations of naïve semantic theory; or in the way that—in my opinion—Zeno's paradox of motion clearly shows us that there's something fundamentally amiss in the naïve idea that the natural spacetime world bottoms out in something that essentially has the structure of the mathematical continuum and that real motion consists in the translation of a point or body across that mathematical continuum, and correspondingly, that there's also something fundamentally amiss with the popular solution to Zeno's paradox of motion that uncritically assumes the truth of that naïve idea, and then appeals to the notion of a "supertask"—i.e., a manifestly real act, operation, or terminating procedure, composed of infinitely many steps, that's nevertheless completed in a finite amount of time (Manchak and Roberts, 2022)—and the calculus of infinitesimals (Hanna, 2023d).

On the contrary, I think that the existence of phenomenological paradoxes of illegibility clearly shows us *the proper bounds or limits of all logic, and indeed of every formal science including mathematics and computer science, and also of every natural science including*

physics, chemistry, and biology: namely, that, on pain of paradox, no logical theory, no other formal science, and no natural science can explicitly or implicitly deny or rule out that it itself must be created or produced as, must be laid-out or presented as, and must also be communicated or disseminated as, *a set of legible texts,* and that it therefore presupposes and uses the rational, conscious, intentional, and intersubjective act or process of reading. So the logic of legibility lays down a set of boundary or limiting conditions for *all* the formal and natural sciences. This necessary proper bounding or limiting fact about all the formal and natural sciences, in turn, is intimately bound up with the necessary facts about human theoretical rationality that I've called *the ratiocentric predicament*, *the psychocentric predicament*, and *the-philosophy-of-reading-as-first-philosophy* (Hanna, 2023e, 2023f, 2023g).

Ultimately, moreover, all of these facts would have to be directly and irreducibly incorporated into *a soft science of the mind* (Hanna, 2023f) that holistically and systematically triangulates (i) evidence from empirical psychology, including evidence from cognitive neuroscience, along with evidence from other formal or natural sciences, (ii) phenomenological evidence, and (iii) classical philosophical evidence—a science of the mind that doesn't *yet* exist, but which certainly *ought* to exist: *the science of reading*.[2]

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REFERENCES

(Boolos and Jeffrey, 1989). Boolos, G. and Jeffrey, R. *Computability and Logic*. 3rd edn., Cambridge: Cambridge Univ. Press.

(Cantini and Bruni, 2021). Cantini, A. and Bruni, R. "Paradoxes and Contemporary Logic." In E.N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy.* Fall 2021 Edition. Available online at URL = <https://plato.stanford.edu/archives/fall2021/entries/paradoxes-contemporary-logic/>.

(Gödel, 1931/1967). Gödel, K. "On Formally Undecidable Propositions of Principia Mathematica and Related Systems." In J. van Heijenoort (ed.), *From Frege to Gödel.* Cambridge MA: Harvard Univ. Press. Pp. 596-617.

(Hanna, 2006). Hanna, R. *Rationality and Logic*. Cambridge: MIT Press. Available online in preview at URL = <https://www.academia.edu/21202624/Rationality_and_Logic>.

(Hanna, 2015). Hanna, R. *Cognition, Content, and the A Priori: A Study in the Philosophy of Mind and Knowledge.* THE RATIONAL HUMAN CONDITION, Vol. 5. Oxford: Oxford Univ. Press. Also available online in preview at URL = <https://www.academia.edu/35801833/The_Rational_Human_Condition_5_Cognition_Content_and_the_A_Priori_A_Study_in_the_Philosophy_of_Mind_and_Knowledge_OUP_2015_>.

(Hanna, 2023a). Hanna, R. "Caveat Lector: From Wittgenstein to The Philosophy of Reading." Unpublished MS. Available online at URL = <https://www.academia.edu/103950659/Caveat_Lector_From_Wittgenstein_to_The_Philosophy_of_Reading_October_2023_version_>.

(Hanna, 2023b). Hanna, R. "Are There Some Legible Texts That Even The World's Most Sophisticated Robot Can't Read?" Unpublished MS. Available online at URL = <https://www.academia.edu/95866304/Are_There_Some_Legible_Texts_That_Even_The_Worlds_Most_Sophisticated_Robot_Cant_Read_January_2023_version_>.

(Hanna, 2023c). Hanna, R. "The Myth of Artificial Intelligence and Why It Persists." Unpublished MS. Available online at URL = <https://www.academia.edu/101882789/The_Myth_of_Artificial_Intelligence_and_Why_It_Persists_May_2023_version_>.

(Hanna, 2023d). Hanna, R. "How To Solve Zeno's Paradox of Motion *Without* Supertasks." Unpublished MS. Available online at URL = <https://www.academia.edu/49014434/How_To_Solve_Zenos_Paradox_of_Motion_Without_Supertasks_August_2023_version_>.

(Hanna, 2023e). Hanna, R. "The Circularity of Human Rationality." Unpublished MS. Available online at URL = <https://www.academia.edu/106083565/The_Circularity_of_Human_Rationality_August_2023_version_>.

(Hanna, 2023f). Hanna, R. "The Psychocentric Predicament, The Impossibility of Any and Every Hard Science of Consciousness, and Soft Sciences of the Mind." Unpublished MS. Available online at URL = <https://www.academia.edu/106873375/The_Psychocentric_Predicament_The_Impossibility_of_Any_and_Every_Hard_Science_of_Consciousness_and_Soft_Sciences_of_the_Mind_October_2023_version_>.

(Hanna, 2023g). Hanna, R. "The Philosophy of Reading as First Philosophy." Unpublished MS. Available online at URL = <https://www.academia.edu/107390679/The_Philosophy_of_Reading_as_First_Philosophy_September_2023_version_>.

(Manchak and Roberts, 2022). Manchak, J.B. amd Roberts, B.W. "Supertasks." In E.N. Zalta (ed.), *Stanford Encyclopedia of Philosophy*. Summer Edition. Available online at URL = <http://plato.stanford.edu/entries/spacetime-supertasks/>.

(Sorensen, 2022). Sorensen, R. "Epistemic Paradoxes." In E.N. Zalta (ed.) The Stanford Encyclopedia of Philosophy. Spring Edition. Available online at URL = <https://plato.stanford.edu/archives/spr2022/entries/epistemic-paradoxes/>.

(Valentino, 2020). Valentino, M. "Disegno ambiguo e sagace." *ResearchGate*. Available online at URL = <https://www.researchgate.net/publication/352757137_Disegno_ambiguo_e_sagace/figures?lo=1>.