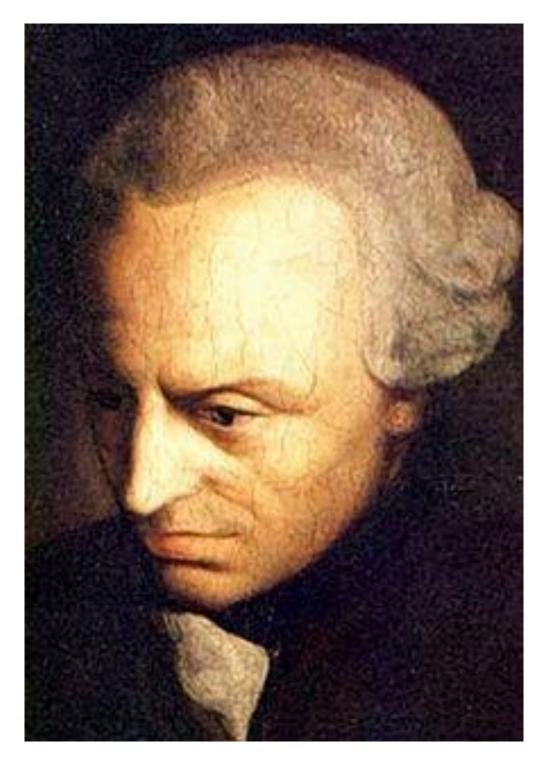
# The Limits of Sense and Reason: A Line-By-Line Critical Commentary on Kant's *Critique of Pure Reason*

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### The Structure of the Critique of Pure Reason

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#### CPR TEXT Avii-ix/GW99 Preface to the First (A) Edition

#### (Avii) PREFACE<sup>a</sup>

Human reason has the peculiar fate in one species of its cognitions that it is burdened with questions which it cannot dismiss, since they are given to it as problems<sup>b</sup> by the nature of reason itself, but which it also cannot answer, since they transcend every capacity<sup>c</sup> of human reason.

Reason falls into this perplexity through no fault of its own. It begins from principles whose use is unavoidable in the course of experience and at the same time sufficiently warranted by it. With these principles it rises (as its nature also requires) ever higher, to more (Aviii) remote conditions. But since it becomes aware in this way that its business must always remain incomplete because the questions never cease, reason sees itself necessitated to take refuge in principles that overstep all possible use in experience, and yet seem so unsuspicious that even ordinary common sense agrees with them. But it thereby falls into obscurity and contradictions, from which it can indeed surmise that it must somewhere be proceeding on the ground of hidden errors; but it cannot discover them, for the principles on which it is proceeding, since they surpass the bounds of all experience, no longer recognize any stone of experience. The battlefield of these endless controversies is called **metaphysics**.

There was a time when metaphysics was called the **queen** of all the sciences, and if the will be taken for the deed, it deserved this title of honor, on account of the preeminent importance of its object. Now, in accordance with the fashion of the age, the queen proves despised on all sides; and the matron, outcast and forsaken, mourns (Aix) like Hecuba: *Modo maxima rerum, tot generis natisque potens - nunc trahor exul, inops -* Ovid, *Metamorphoses.*<sup>d</sup>

<sup>*a*</sup> As in the first edition. Kant wrote a new preface for the second edition, given below [GW: 106-124]. <sup>*b*</sup> *aufgegeben* <sup>*c*</sup> *Vermögen* <sup>*d*</sup> "Greatest of all by race and birth, I now am cast out, powerless" (Ovid, Metamorphoses 13:508-10).

#### COMMENTARY ON Avii-ix/GW99 Preface to the First (A) Edition

The first or A edition Preface was replaced in the second or B edition by a completely new Preface.

The B Preface more fully elaborates the basic ideas of the A Preface, and places them in a larger philosophical context, presumably in order to respond to various criticisms and also to pre-empt (what Kant took to be) various serious critical misunderstandings of the CPR that had appeared in print between 1781 and 1787.

This *especially* includes the critical accusation that Kant's real metaphysics, transcendental idealism, is nothing but

either (i) a version of Cartesian dream skepticism about our perceptual knowledge of the existence and/or nature of the external world (which Kant informatively calls "problematic idealism," but nowadays is often called, somewhat less informatively, "veil-of-perception skepticism"),

or (ii) a version of Berkeley's skeptical subjective idealism (which Kant also informatively calls "dogmatic idealism").

Kant expanded or even significantly modified his views on some central issues between the A and B editions.

So in addition to unpacking the basic philosophical content of the two Prefaces, I will also highlight the most philosophically important differences between them.

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As we've seen, the fundamental twofold theme of the CPR is:

(i) the critical rejection of classical metaphysics, especially including classical Rationalist metaphysics, but also including various versions of dualism and materialism or physicalism—according to Kant, metaphysical materialism must be sharply distinguished from classical Empiricism, in the sense that it is logically possible to accept materialism/physicalism and reject classical Empiricism, and conversely, and

(ii) the revolutionary replacement of classical metaphysics by an essentially "human, all too human" or anthropocentric, mitigated rationalist, real metaphysics: transcendental idealism.

And as we've also seen, The Motto in the B edition conveys this fundamental twofold theme epigrammatically.

The A Preface, however, conveys this same fundamental theme by means of a rhetorically quite effective and semantically pregnant *framing metaphor* based on the tragic plight of Hecuba.

Hecuba was a mythic queen of Troy, wife of King Priam, and the mother of Hector, Cassandra, and some forty-eight other children (some of them, presumably, the offspring of Priam's concubines), many of whom died violent deaths.

Hecuba was enslaved by the Achaeans after the Fall of Troy.

Kant cites her tragic lament, as movingly portrayed in Ovid's Metamorphoses:

Greatest of all by race and birth, I am now cast out, powerless.<sup>1</sup>

There are other equally moving portrayals in Euripides's *Hecuba* and *The Trojan Women*, and in Dante's 16<sup>th</sup> century *Inferno*, the first part of *The Divine Comedy*.

Significantly, Dante's description of Hecuba has her ultimately descending into madness, and barking like a dog.<sup>2</sup>

Kant's framing metaphor aligns Hecuba's plight with the tragic career of classical Rationalist metaphysics, the erstwhile "queen of all the sciences."

The truly memorable opening line of the CPR ("Reason has the peculiar fate in one species of its cognition that it burdened with questions that it cannot dismiss...") tells us that there is an innate cognitive capacity, faculty, or power (*Vermögen*), namely reason or *Vernunft*, which is partially *constitutive* of human cognition, but at the same time this very faculty of reason is the inherent source of insoluble difficulties for human cognition.

More precisely, the faculty of reason in its human involvement naturally presents certain philosophical problems that lead inevitably to corresponding questions, which in turn simply cannot be answered, even in principle, since the answers would require a kind of knowledge that inherently transcends the scope of human reason itself.

So in other words, the very cognitive faculty that, according to classical Rationalism, is supposed to be the strict determiner of the content, truth, and justification of necessary a priori judgments, by a wholly tragic reversal of fortune, turns out to be the primitive source of its own rational self-stultification, or cognitive suicide—in effect, a self-inflicted descent into barking madness, like poor Hecuba. How the philosophically mighty have fallen.

Kant then provides a psychologically-oriented *meta*philosophical diagnosis of the logico-metaphysical and pragmatic self-stultification, or *cognitive suicide*, of the faculty of reason.

But *it's not reason's fault*—if by "it's reason's fault," it is meant that human reason had *mistakenly or wrongly chosen* to stultify itself.

On the contrary, *it is built into the very nature of human reason* that it cognizes and follows principles that are adequately warranted by their application to human experience, yet the recursive application of these same principles—their consistent, constructive application to the results of previous applications of the very same principles—leads human reason beyond the original data of human experience, and into applications that extend unrestrictedly beyond experience, and ultimately into "obscurity and contradictions" (*Dunkelheit und Widersprüche*).

Precisely what kind of obscurity, and precisely what kind of contradictions?

Here there's an essential anticipation by Kant of the logico-mathematical phenomenon of what contemporary philosophers of logic and mathematics call *impredicativity*, and even more precisely, of what I will call *vicious impredicativity*.

Impredicativity is the construction or definition of sets or totalities of objects in terms of, or by reference to, those very totalities themselves.

E.g., the set or totality of all things that are not bachelors<sup>3</sup> is itself *not a bachelor*, and thus belongs to the membership of that very set or totality.

On the other hand, the set or totality of all bachelors is *not* itself a member of the set or totality of bachelors.

So some sets or totalities are members of themselves, and some sets or totalities are not members of themselves.

Let's call the self-membership of the set or totality of all things that are not bachelors an instance of *benign impredicativity*, and let's call the rule which says that sets or totalities may permissibly be constructed or defined by impredicative operations, *The Principle of Benign Impredicativity*.

Correspondingly, the construction or definition of sets or totalities according to The Principle of Benign Impredicativity is perfectly logically and mathematically legitimate.

This in turn leads to the corresponding notion of a *well-founded* or *phenomenal* set or totality:

A set or totality is well-founded or phenomenal if and only if

either (i) every element of its membership (leaving aside the empty set<sup>4</sup>) is an actual or possible object of human experience,

or (ii) all its iterative self-including constructions necessarily presuppose that every element of its first-order membership (leaving aside the empty set) is an actual or possible object of human experience.

By sharp contrast, what I'll call *vicious impredicative reasoning* is the iterative selfincluding construction of higher-order set or totalities whose first-order membership consists of *indefinitely many* elements that are *other than* actual or possible objects of human experience.<sup>5</sup>

Sets or totalities created by vicious impredicative reasoning are what I'll call *ill-founded* or *noumenal* sets or totalities.

Vicious impredicativity, and its ill-founded or noumenal set or totalities, yield *antinomies*, aka *paradoxes*, aka *hyper-contradictions*, aka *dialetheias*: propositions such that, logically necessarily, they are true if and only if they are false.

Antinomies, paradoxes, hyper-contradictions, or dialetheias are such that, *logically necessarily* they are both true *and* false: hence they are "truth-value gluts."

Correspondingly, Kant argues in the Dialectic of Pure Reason that *no* well-founded or phenomenal set or totality is antinomous/paradoxical/hyper-contradictory/dialetheic, whereas *every* ill-founded or noumenal set or totality is antinomous/paradoxical/hyper-contradictory/dialetheic.<sup>6</sup>

These points also enable us to frame a Kantian logico-mathematical distinction between two sharply different types of *universal* sets or totalities: *absolutely universal* sets or *totalities* and *restrictedly universal* sets or *totalities*.

Absolutely universal sets or totalities include, e.g., Kant's *omnitudo realitatis* (CPR A576/B604), Russell's set *w* of all sets that are not members of themselves,<sup>7</sup> and Cantor's universal set *C*, corresponding to the greatest cardinal number.<sup>8</sup>

Absolutely universal sets or totalities are *vicious impredicative sets or totalities*, exactly because they are ill-founded and noumenal, and yield antinomies/paradoxes/hyper-contradictions/dialetheias.

By sharp contrast, restrictedly universal sets or totalities include Kant's transcendentally ideal/empirically real space and time as what he calls *infinite given magnitudes* (CPR A25/B39-40, A32/B48-49) and Cantor's *transfinite* sets—e.g., the set of real numbers—as constructed by the power set operation on denumerably infinite sets.

All such sets or totalities are *benign impredicative sets or totalities*, exactly because, although they are impredicatively constructed by virtue of *including everything in some infinite class of things, including themselves*, they are also well-founded and phenomenal, as well as being logically non-contradictory.

The most philosophically famous—and, for my purposes in LSR, the historically most directly relevant—example of vicious impredicativity is "The Contradiction" discovered by Russell in Frege's *Basic Laws of Arithmetic*, in basic law V of Frege's logical system, *the unrestricted comprehension principle for naïve set theory*, which says that any well-formed predicate intension uniquely determines a well-formed concept, whose extension is a well-formed set.

For Frege, Russell's discovery had not only logically catastrophic results, but almost literally Hecuba-like personal consequences.

As he wrote to Russell:

Your discovery of the contradiction has surprised me beyond words and, I should like to say, left me thunderstruck because it has rocked the ground on which I mean to build arithmetic.... It is all the more serious as the collapse of my law V seems to undermine not only the foundations of my arithmetic but the only possible foundations of arithmetic as such.<sup>9</sup>

Russell's discovery was not only *logically catastrophic* and *personally tragic* for Frege: it was also, from the prospective standpoint of Kant's CPR, *metaphysically ironic*.

This is because Frege's goal in *Basic Laws* is precisely to provide an explanatory and ontolological reduction of arithmetic to logic and thereby, by showing that arithmetic truths are analytic truths, refute *Kant's* thesis that arithmetic is synthetic a priori.

Nevertheless, as we've just seen, Frege's attempt to resuscitate the project of classical Rationalist metaphysics *against Kant*, in the guise of Logicism, ended in *Dunkelheit und Widersprüche*.

Here, in a nutshell, is how The Contradiction happened, how Logicism ended, and how Kant was vindicated by the history of logic and mathematics.

In order to reduce arithmetic to logic, Frege presupposed and used naïve set theory as the *reducing* theory, and characterized numbers as sets of all sets whose membership can be put into one-to-one correspondence with each other (equinumerosity).

We've seen that by an application of The Principle of Benign Impredicativity to the objects of ordinary experience, there are some sets or totalities that *are* members of themselves, like the set or totality of all things that are not bachelors, and that there are other sets or totalities that are *not* members of themselves, like the set or totality of bachelors.

But now, by a recursive process that clearly extends beyond the scope of human experience, what about the set or totality consisting of *all* sets or totalities that are not members of themselves?

Call this non-experiential set or totality *w*.

If *w* is a member of itself, then *w* is not a member of itself.

But if *w* is not a member of itself, then *w* is a member of itself.

So we discover, to our "human, all too human" rational dismay, that *w* is a member of itself if and only if it's not a member of itself.

In other words, *Dunkelheit und Widersprüche*!, and we directly bear witness to pure reason's self-inflicted descent into barking madness, by virtue of its construction of *an absolute, ill-founded, noumenal totality*.

This is a particularly crisp and vivid example of what Kant later in the CPR calls *the antinomy of pure reason,* the discovery of which in 1766, as we saw above, was the third

basic source of Kant's revolutionary anthropocentric turn to the mitigated rationalism and real metaphysics of transcendental idealism.

In 1903 and 1908 Russell called instances of the antinomy of pure reason *The Contradictions*,<sup>10</sup> but *like* Frege and *unlike* Kant, Russell also refused to trace their generation to the innate constitution of human reason itself and its natural and "human, all too human" psychological tendency to engage in vicious impredicative reasoning.

Russell's refusal, like Frege's similar refusal, stemmed directly from the explicitly anti-Kantian metaphysical and epistemic commitments of Russell's Logicism:

[T]he Kantian view . . . asserted that mathematical reasoning is not strictly formal, but always uses intuitions, **i.e.** the **à priori** knowledge of space and time. Thanks to the progress of Symbolic Logic, especially as treated by Professor Peano, this part of the Kantian philosophy is now capable of a final and irrevocable refutation.<sup>11</sup>

*Ever since I abandoned the philosophy of Kant . . . I have sought solutions of philosophical problems by means of analysis; and I remain firmly persuaded . . . that only by analysing is progress possible.*<sup>12</sup>

Just as Frege's Logicism had failed because of The Contradictions' emergence in basic law V, so too Russell's Logicism failed for a somewhat different—although, in the end, not *fundamentally* different—reason.

In 1931, Kurt Gödel proved two seminal *incompleteness* theorems, which jointly show that all classical second-order logical systems, like Russell and Whitehead's system in *Principia Mathematica*, that contain (enough of) Peano's five axioms for arithmetic, must also contain logically unprovable sentences (in effect, self-referring versions of The Contradiction formally equivalent to the Liar Paradox) and are therefore

not only (i) incomplete (that is, not all of their tautologies are theorems), but also

but also (ii) consistent if and only if the ground of truth for such systems is outside the system itself.<sup>13</sup>

Gödel's seminal incompleteness results are formally highly analogous to Kant's dialectical logical analysis of the antinomy of pure reason, which shows that classical metaphysics, especially including classical Rationalist metaphysics, logically entails antinomies *if and only if* it's assumed that there is no fundamental ontological difference between appearances or phenomena, and things-in-themselves or noumena.

In other words, for Kant, classical metaphysics, especially including classical Rationalist metaphysics, is

not only (i) incomplete (because its basic theses lead to antinomies/paradoxes/hyperontradictions/dialetheias),

but also (ii) consistent if and only if its ground of truth includes a fundamental ontological distincion between phenomena and noumena.

Clearly, then, there's some deep and essential logico-metaphysical connection, discovered by Kant, and later re-discovered by Gödel, between

(i) collapsing the fundamental ontological difference between phenomena and noumena,

- (ii) vicious impredicative reasoning,
- (iii) absolute, ill-founded, noumenal totalities,
- (iv) The Contradictions,
- (v) Incompleteness, and
- (vi) anti-Logicism.

But in any case, both Frege's and Russell's versions of logicism ultimately failed, precisely because of their inability to control the logically explosive power of (ii) to (v): hence it's more than merely reasonable to seek a deeper explanation for these cataclysmic logical explosions in the metaphysical black hole created by confusing noumena and phenomena.

It's now known, however, that Peano arithmetic is logically provable in classical second-order logic together with a conceptual replacement for the Frege's attempted reduction of numbers to sets, known as *Hume's Principle*:

The number of Fs = the number of Gs if and only if there are exactly as many Fs as there are Gs.

This is the logico-metaphysical foundation of the late 20<sup>th</sup> century project of *Neo-Logicism*.<sup>14</sup>

What remains an importantly open logico-metaphysical question, however, is whether second-order logic and Hume's Principle are *analytic* or *synthetic a priori*.

If, as I believe, Hume's Principle is indeed synthetic priori, then Kant was right after all, hence he was vindicated by the history of logic and mathematics; and as a consequence, both Logicism and Neo-Logicism, just like classical and especially classical Rationalist metaphysics, were and are tragically mistaken.

Nevertheless, the whole debate concerning The Contradictions, Incompleteness, and the philosophical fate of Logicism and Neo-Logicism, has been made almost impossibly difficult to discuss dispassionately in or out of print, or satisfactorily resolve, by the sad historico-philosophical fact that even now, 238 years after the original publication of the CPR, there's neither a generally accepted theory of the analytic-synthetic distinction, nor a generally accepted theory of the nature and status of logic.<sup>15</sup>

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To summarize the philosophical plot up to here.

Kant's first two main points in the A Preface, to be reprised and worked out in full detail later in the chapter entitled "The Antinomy of Pure Reason," are these:

(i) classical metaphysics, especially including classical Rationalist metaphysics, is inherenly *capable* of generating an antinomy of pure reason precisely because the innate constitution of the faculty of human reason provides for a natural psychological tendency to engage in transcendent impredicative reasoning, and

(ii) the faculty of human reason on its own, without appealing to any *other* basic human cognitive faculty—for example, the faculty of sensibility—is also inherently *incapable* of comprehending what has gone wrong in its reasoning processes when it generates an antinomy of pure reason.

Indeed, Kant even goes so far here as to *identify* metaphysics with reasoning that satisfies these two conditions:

The battlefield of these endless controversies is called metaphysics. (CPR Aviii)

Now it's bad enough, and certainly philosophically tragic, that classical Rationalist metaphysics commits cognitive suicide by means of its vicious impredicative reasoning,

its absolute, ill-founded, noumenal totalities, and its antinomies/paradoxes/hypercontradictions/dialetheias.

But its inevitable self-ignorance about how and why this is happening is *also* a direct violation of the original Socratic philosophical imperative to "know thyself."

Hence at this point, one might well think:

"Well, so much the worse for human reason, and its supposedly innate faculty for infallible clear and distinct rational intuition!"

And like Hume in the first *Enquiry*, one might well think that, as a consequence, we should simply *burn* all classical Rationalist metaphysics *at the stake*:

When we run over libraries, persuaded of these [Empiricist] principles, what havoc must we make? If we take in our hand any volume; of divinity or school metaphysics, for instance; let us ask, **Does it contain any abstract reasoning concerning quantity or number?** No. **Does it contain any experimental reasoning concerning matter of fact and existence?** No. Commit it then to the flames: for it can contain nothing but sophistry and illusion.<sup>16</sup>

But although in 1771 or 1772 Kant was indeed philosophically awakened from his dogmatic slumbers and enlightened for the third time in a six-year span, and this time by Hume, nevertheless, on the other hand, he was never *mesmerized* by Hume's skeptical Empiricism.

So the very fact that human reason has this "peculiar fate" (*besondere Schicksal*) driving it towards cognitive self-annihilation, according to Kant, also ultimately drives self-critical human reason through and beyond Hecuba's tragedy towards a tragic rational *catharsis* in Aristotle's sense in the *Poetics*, that is, towards a purging of otherwise harmful emotions for the sake of self-knowledge.

And by means of this process of rational catharsis, as self-clarifying, self-critical, self-knowing metaphysicians and knowers, we rid ourselves of the very *need* to transcend human experience, and fully accept our "human, all too human" finitude.

According to Kant, we thereby become *mitigated rationalists* and *real metaphysicians*, not Humean Empiricist *mitigated skeptics* and *anti-metaphysicians* about human reason.

I'll have more to say about this fundamental contrast between Kant's mitigated rationalism and real metaphysics, on the one hand, and Hume's Empiricist mitigated skepticism and anti-metaphysics on the other, in later installments of this commentary.

## NOTES

<sup>1</sup> Ovid, *Metamorphoses*, trans. D. Raeburn (New York/London: Random House/Penguin, 2004), 13: 508ff.

<sup>2</sup> Dante Alighieri, *The Inferno*, in Dante Alighieri, *The Divine Comedy*, 3 vols., trans. D.L. Sayers (Harmondsworth, Middlesex UK: Penguin, 1955), vol. 1, canto XXX, 13-20.

<sup>3</sup> Kant distinguishes carefully between two types of negation: (i) classical propositional or predicate negation (e.g., "The cat is not on the mat," and "Cats are not dogs") and (ii) what he calls "infinite negation," which yields an indefinitely large a set or totality that is *the actual-world and possible-worlds complement* to a given definite set or totality (e.g., "God is a non-dog" and "Gods are non-dogs") (CPR A70-73/B95-98).

<sup>4</sup> *The empty set,* or what Kant calls "empty intuition without an object," as we will see later, is a special representation of pure intuition that is also minimally contained in every non-empty first-order set or totalitity whatsoever (CPR A290-292/B346-349).

<sup>5</sup> See note 3 above. Classical propositional or predicate negation corresponds to benign impredicative reasoning and well-founded, phenomenal sets or totalities, whereas infinite negation corresponds to vicious impredicative reasoning and absolute, ill-founded, noumenal totalities.

<sup>6</sup> The occurrence of antinomy/paradox/hyper-contradiction/dialetheia within a logical system does not in fact indicate its logical *incoherence*, but rather only its logical *deviance*. It remains rationally possible to accept and indeed affirm the logical existence of antinomy/paradox/hyper-contradiction/dialetheia, simply by means of constructing logically coherent, non-classical, "paraconsistent" systems. More precisely, a logical system is paraconsistent if and only if it contains some antinomous/paradoxical/hyper-contradictory/dialethethic theorems *plus* an axiom that prevents explosion. See, e.g., S. Haack, *Deviant Logic, Fuzzy Logic* (Chicago, IL: Univ. of Chicago Press, 1996); and G. Priest, *An Introduction to Non-Classical Logic* (Cambridge: Cambridge Univ. Press, 2001). In effect, then, Kant's Dialectic of Pure Reason is a study in deviant logic that's driven by *metaphysical and epistemic radical agnosticism*. Hegel's *Logic*, by contrast, is a study in deviant logic that's driven by *metaphysical and epistemic megalomania*.

<sup>7</sup> See B. Russell, "Mathematical Logic as Based on the Theory of Types," in B. Russell, *Logic and Knowledge* (London: Unwin Hyman, 1956), pp. 59-102, at p. 59.

<sup>8</sup> See, e.g., *Wikipedia*, "Cantor's Paradox," available online at URL = <<u>https://en.wikipedia.org/wiki/Cantor%27s\_paradox</u>>.

<sup>9</sup> R. Monk, *Bertrand Russell: The Spirit of Solitude* (London: Jonathan Cape, 1996), p. 153.

<sup>10</sup> See B. Russell, *The Principles of Mathematics* (2<sup>nd</sup> edn.; New York: W.W. Norton, 1996); and Russell, "Mathematical Logic as Based on the Theory of Types."

<sup>11</sup> Russell, *Principles of Mathematics*, p. 4.

<sup>12</sup> Russell, My Philosophical Development (London: Allen & Unwin, 1959), pp. 14-15.

<sup>13</sup> K. Gödel, "On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems," in J. van Heijenoort (ed.), *From Frege to Gödel* (Cambridge, MA: Harvard Univ. Press, 1967), pp. 596-617.

<sup>14</sup> See, e.g., C. Wright, *Frege's Conception of Numbers as Objects* (Aberdeen: Aberdeen Univ. Press, 1983); B. Hale, *Abstract Objects* (Oxford: Blackwell, 1987); and B. Hale and C. Wright, *The Reason's Proper Study: Essays Towards a Neo-Fregean Philosophy of Mathematics* (Oxford: Oxford Univ. Press, 2001).

<sup>15</sup> But for an attempt at these, see R. Hanna, *Rationality and Logic* (Cambridge, MA: MIT Press, 2006), also available online in preview, <u>HERE</u>; and R. Hanna, *Cognition, Content, and the A Priori: A Study in the Philosophy of Mind and Knowledge* (THE RATIONAL HUMAN CONDITION, Vol. 5) (Oxford: Oxford Univ. Press, 2015), chs. 4-5, also available online in preview, <u>HERE</u>.

<sup>16</sup> D. Hume, *Enquiries Concerning Human Understanding and Concerning the Principles of Morals* (Oxford: Oxford Univ. Press, 1975), section XII, part III, p. 165.